Optimization of functionals

Regular course

Department of Mathematics, Cinvestav

Description

This course is an introduction to three of the main techniques that are used to minimize functionals: calculus of variations, Pontryagin's principle, and dynamic programming. Some applications and computational algorithms are also studied.

Contents

1. Basics of optimization in $\mathbb{R}^n$ [6]
   - 1.1. Necessary conditions
   - 1.2. Convex sets and convex functions
   - 1.3. The Lagrange multiplier method
   - 1.4. Inequality constraints
2. Lower semicontinuous functions in metric spaces [6, 2]
   - 2.1. Existence of minimizers. Coercive functions
   - 2.2. Ekeland's variational principle
   - 2.3. Fritz John necessary conditions
3. Calculus of variations [1, 10]
   - 3.1. The simplest problem of the calculus of variations
   - 3.2. The Euler-Lagrange equation
   - 3.3. Sufficient conditions
4. Correspondences [2]
   - 4.1. Continuity of correspondences
   - 4.2. Berge theorem
   - 4.3. Measurable correspondences
   - 4.4. The theorem of Kuratowski and Ryll-Nardzewski
   - 4.5. Filippov's implicit theorem
5. Optimal control of ordinary differential equations [3, 4, 8]
   - 5.1. Existence of solutions
   - 5.2. Ekeland's metric
   - 5.3. Pontryagin's principle
   - 5.4. Sufficient conditions
6. Dynamic programming [7]

6.1. The algorithm of dynamic programming

6.2. The Hamilton-Jacobi-Bellman equation

6.3. The relationship to Pontryagin's principle

7. Additional topics [5, 9]

7.1. Applications

7.2. Computational algorithms

Referencias

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[4] I. Ekeland, *On the variational principle*, J. Math. Anal. Appl., 47 (1974), pp. 324–353.

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[7] O. Hernández-Lerma, L. R. Laura-Guarachi, S. Mendoza-Palacios, and D. González-Sánchez, *An introduction to optimal control theory*, Springer, Cham, 2023.

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