

Optimization of functionals

Regular course
Department of Mathematics, Cinvestav

DESCRIPTION

This course is an introduction to three of the main techniques that are used to minimize functionals: calculus of variations, Pontryagin's principle, and dynamic programming. Some applications and computational algorithms are also studied.

CONTENS

1. Calculus of variations [1, 10]
 - 1.1. Introduction: the brachistochrone problem
 - 1.2. The simplest problem of the calculus of variations
 - 1.3. The Euler-Lagrange equation
 - 1.4. Convex functions. Sufficient conditions
2. Lower semicontinuous functions in metric spaces [6, 2]
 - 2.1. Existence of minimizers
 - 2.2. Ekeland's variational principle
 - 2.3. Fritz John necessary conditions
3. Correspondences [2]
 - 3.1. Continuity of correspondences
 - 3.2. Berge theorem
 - 3.3. Measurable correspondences
 - 3.4. The theorem of Kuratowski and Ryll-Nardzewski
 - 3.5. Filippov's implicit theorem
4. Optimal control of ordinary differential equations [3, 4, 8]
 - 4.1. Existence of solutions
 - 4.2. Ekeland's metric
 - 4.3. Pontryagin's principle
 - 4.4. Sufficient conditions
5. Dynamic programming [7]
 - 5.1. The algorithm of dynamic programming
 - 5.2. The Hamilton-Jacobi-Bellman equation
 - 5.3. The relationship to Pontryagin's principle
6. Additional topics [5, 9]
 - 6.1. Applications
 - 6.2. Computational algorithms

Referencias

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- [4] I. EKELAND, *On the variational principle*, J. Math. Anal. Appl., 47 (1974), pp. 324–353.
- [5] D. GRASS, J. P. CAULKINS, G. FEICHTINGER, G. TRAGLER, AND D. A. BEHRENS, *Optimal control of nonlinear processes*, Springer-Verlag, Berlin, 2008.
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- [8] A. D. IOFFE AND V. M. TIHOMIROV, *Theory of extremal problems*, North-Holland Publishing Co., Amsterdam-New York, 1979. Translated from the Russian by Karol Makowski.
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